# Intrusion Detection: Base Rate Fallacy 

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## Lecture 8-1

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Testing Outcomes

S: Patient is Sick (has the disease)
$S \quad \neg S$

R: Test Result is positive

| $R \wedge S$ | $\mathrm{R} \wedge \neg \mathrm{S}$ |
| :---: | :---: |
| True positive | False positive |
| $\neg \mathrm{R} \wedge \mathrm{S}$ | $\neg \mathrm{R} \wedge \neg \mathrm{S}$ |
| False negative | True negative |

Testing Outcomes

R: Test Result is positive Alarm is raised

| R | $R \wedge S$ | $\mathrm{R} \wedge \neg \mathrm{S}$ |
| :---: | :---: | :---: |
|  | True positive | False positive |
|  | $\neg \mathrm{R} \wedge \mathrm{S}$ | $\neg \mathrm{R} \wedge \neg \mathrm{S}$ |
| $\neg \mathrm{R}$ | False negative | True negative |

Malware Detection Techniques


Testing Outcomes

S: Patient is Sick (has the disease)
$S \quad \neg S$

R: Test Result is positive

| $R \wedge S$ | $\mathrm{R} \wedge \neg \mathrm{S}$ |
| :---: | :---: |
| True positive | False positive |
| $\neg \mathrm{R} \wedge \mathrm{S}$ | $\neg \mathrm{R} \wedge \neg \mathrm{S}$ |
| False negative | True negative |

Testing Outcomes


## Estimating $\mathrm{P}(\mathrm{R} \mid \mathrm{S})$ etc



1000 not sick

Test R is positive 1980

Test R is negative

20


Test R is negative 990 estimate $\quad \mathrm{P}(\mathrm{R} \mid \mathrm{S})=0.99$

$$
P(\neg R \mid S)=0.01
$$

$$
P(R \mid \neg S)=0.01
$$

$$
P(\neg R \mid \neg S)=0.99
$$



Coincidentally equal

## Estimating $\mathrm{P}(\mathrm{R} \mid \mathrm{S})$ etc

Test R is positive 1980


Test R is negative

20

1000 not sick


Test R is negative

970
estimate

$$
P(R \mid S)=0.99
$$

$$
\mathrm{P}(\neg \mathrm{R} \mid \mathrm{S})=0.01 \quad \mathrm{P}(\mathrm{R} \mid \neg \mathrm{S})=0.03
$$

$$
P(\neg \mathrm{R} \mid \neg \mathrm{S})=0.97
$$



In general will not be equal

## Base-Rate Fallacy

S: Patient is Sick (has the disease)


Columns must total 1

Rows must total between 0 and 2

We will continue with these numbers


R: Test Result is positive

These probabilities can be empirically estimated

S: Patient is Sick
(has the disease)

S ᄀS

| R | $R \wedge S$ <br> True positive $P(R \mid S)=0.99$ | $R \wedge \neg S$ <br> False positive $P(R \mid \neg S)=0.01$ |
| :---: | :---: | :---: |
|  |  |  |
| is positive | $\neg \mathrm{R} \wedge \mathrm{S}$ | $\neg \mathrm{R} \wedge \neg \mathrm{S}$ |
| $\neg \mathrm{R}$ | False negative | True negative |
| These probabilities can be empirically | $\mathrm{P}(\neg \mathrm{R} \mid \mathrm{S})=0.01$ | $\mathrm{P}(\neg \mathrm{R} \mid \neg \mathrm{S})=0.99$ |

Real Interest


## Bayes' Theorem

$\Rightarrow \mathrm{P}(\mathrm{S} \mid \mathrm{R})=$
$(P(S) \times P(R \mid S)) /$
$(P(S) \times P(R \mid S)+P(\neg S)) \times P(R \mid \neg S))$
$>P(\neg S \mid R)=1-P(S \mid R)$
$>\mathrm{P}(\mathrm{S} \mid \neg \mathrm{R})=$
$(P(S) \times P(\neg R \mid S)) /$
$(P(S) \times P(\neg R \mid S)+P(\neg S)) \times P(\neg R \mid \neg S))$
$\Rightarrow \mathrm{P}(\neg \mathrm{S} \mid \neg \mathrm{R})=1-\mathrm{P}(\mathrm{S} \mid \neg \mathrm{R})$

We will continue with these numbers

## Test Outcomes



## Base Rate

Assume $P(S)=0.0001$ 1 in 10,000 has disease

R: Test Result is positive

These probabilities can be computed by Bayes' theorem if we know P(S)

S: Patient is Sick (has the disease)
$S \quad \neg S$

| $R \wedge S$ | $R \wedge \neg S$ |
| :---: | :---: |
| True positive $P(S \mid R)=0.009804$ | False positive $P(\neg S \mid R)=0.990196$ |
| $\neg \mathrm{R} \wedge \mathrm{S}$ | $\neg \mathrm{R} \wedge \neg \mathrm{S}$ |
| False negative $P(S \mid \neg R)=0.000001$ | True negative $P(\neg S \mid \neg R)=0.999999$ |

Rows must total 1

## False Alarms Predominate!

Assume
$P(S)=0.0001$
1 in 10,000 has disease

| $\mathrm{P}(\mathrm{S} \mid \mathrm{R})$ | requires $\mathrm{P}(\mathrm{R} \mid \neg \mathrm{S})$ |
| :--- | :---: |
| 0.01 | 0.01 |
| 0.09 | 0.001 |
| 0.5 | 0.0001 |
| 0.9 | 0.00001 |
| 0.99 | 0.000001 |




