

# Logic of Authentication

## 1. BAN Logic

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### Source

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These lectures are primarily based on:

- Paul Syverson and Iliano Cervesato, *The Logic of Authentication Protocols*, in R. Focardi, R. Gorrieri (Eds.): *Foundations of Security Analysis and Design*, Lecture Notes in Computer Science, LNCS 2171, Springer-Verlag 2001.

## Protocol 1 (Needham-Schroeder Shared-Key) [NS78]

*Message 1*  $A \rightarrow S : A, B, n_A$

*Message 2*  $S \rightarrow A : \{n_A, B, k_{AB}, \{k_{AB}, A\}_{k_{BS}}\}_{k_{AS}}$

*Message 3*  $A \rightarrow B : \{k_{AB}, A\}_{k_{BS}}$

*Message 4*  $B \rightarrow A : \{n_B\}_{k_{AB}}$

*Message 5*  $A \rightarrow B : \{n_B - 1\}_{k_{AB}}$

Nonces are random unpredictable values generated by a principal and included in messages so that she can tell any messages later received and containing her nonce must have been produced after she generated and sent the nonce.

## BAN Logic

- BAN is a logic of belief.
- In an analysis, the protocol is first idealized into messages containing assertions, then assumptions are stated, and finally conclusions are inferred based on the assertions in the idealized messages and those assumptions.



## The language of BAN

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- In all of these expressions,  $X$  is either a message or a formula.
- As we will see, every formula can be a message, but not every message is a formula.



## The language of BAN

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- *P believes X* :
- *P received X* : message;  
this may require decryption.
- *P said X* :
- *P controls X* :
- *fresh(X)* : (Read 'X is fresh'.)  
X has not been sent in any message prior to current protocol run

## The language of BAN

- $P \leftrightarrow^k Q$  : (Read ‘ $k$  is a good key for  $P$  and  $Q$ ’.)  
 $k$  will never be discovered by any principal but  $P$ ,  $Q$ , or a principal trusted by  $P$  or  $Q$ . (The last case is necessary, since the server often sees, indeed generates,  $k$ .)
- $PK(P, k)$  : (Read ‘ $k$  is a public key of  $P$ ’.)  
The secret key,  $k^{-1}$ , corresponding to  $k$  will never be discovered by any principal but  $P$  or a principal trusted by  $P$ .
- $\{X\}k$  : Short for “ $\{X\}k$  from  $P$ ” (Read ‘ $X$  encrypted with  $k$  (from  $P$ )’.)  
This is the notation for encryption. Principals can recognize their own messages. Encrypted messages are uniquely readable and verifiable as such by holders of the right keys.

## BAN Rules: Message Meaning

$P$  believes  $P \leftrightarrow^k Q$

$P$  received  $\{X\}k$

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$P$  believes  $Q$  said  $X$

“If  $P$  receives  $X$  encrypted with  $k$  and if  $P$  believes  $k$  is a good key for talking with  $Q$ , then  $P$  believes  $Q$  once said  $X$ .”

In applying symmetric keys, there is no explicit distinction between signing and encryption.

## BAN Rules: Message Meaning

*P believes PK(Q, k)*

*P received  $\{X\}k^{-1}$*

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*P believes Q said X*

There is no explicit distinction between signing and encryption. Both are represented by  $\{X\}k$  or  $\{X\}k^{-1}$ . The distinction is implicit in the notation for the key used:  $k$  or  $k^{-1}$ .

## BAN Rules: Nonce Verification

*P believes fresh(X)*

*P believes Q said X*

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*P believes Q believes X*

This rule allows promotion from the past to the present (something said some time in the past to a present belief).

In order to be applied,  $X$  should not contain any encrypted text.

## BAN Rules: Jurisdiction

*P believes Q controls X*

*P believes Q believes X*

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*P believes X*

The jurisdiction rule allows inferences that a principal believes a key is good, even though it is a random string that he has never seen before.

## BAN Rules: Belief Concatenation

*P believes X*

*P believes Y*

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*P believes (X, Y)*

The obvious rules apply to beliefs concerning concatenations of messages/conjunctions of formulae.

Concatenations of messages and conjunctions of formulae are both represented as  $(X, Y)$  in the above rules.

## BAN Rules: Belief Conjunction

$$\frac{P \text{ believes } Q \text{ believes } (X, Y)}{P \text{ believes } Q \text{ believes } X}$$
$$\frac{P \text{ believes } Q \text{ said } (X, Y)}{P \text{ believes } Q \text{ said } X}$$

We do not list all of the rules; we give only a representative sampling.

## BAN Rules: Freshness Conjunction

$$\frac{P \text{ believes } \textit{fresh}(X)}{P \text{ believes } \textit{fresh}(X, Y)}$$

For some inexplicable reason, this is a commonly misunderstood BAN rule. Some try to deny it; others try to assert the converse rule. Be wary of these mistakes.

## BAN Rules: Receiving Rules: Seeing is Receiving

$P$  believes  $P \leftrightarrow^k Q$

$P$  received  $\{X\}k$

$P$  received  $X$

$P$  received  $(X, Y)$

$P$  received  $X$

A principal receiving a message also receives submessages he can uncover.

## BAN Protocol Analysis

1. Idealize the protocol.
2. Write assumptions about the initial state.
3. Annotate the protocol: For each message transmission  $P \rightarrow Q : M$  in the protocol, assert  $Q$  received  $M$ .
4. Use the logic to derive the beliefs held by protocol principals.



## Protocol 1 (Needham-Schroeder Shared-Key) [NS78]

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*Message 2*  $S \rightarrow A : \{n_A, B, k_{AB}, \{k_{AB}, A\}k_{BS}\}k_{AS}$

*Message 3*  $A \rightarrow B : \{k_{AB}, A\}k_{BS}$

*Message 4*  $B \rightarrow A : \{n_B\}k_{AB}$

*Message 5*  $A \rightarrow B : \{n_B - 1\}k_{AB}$

## Idealized Needham-Schroeder Shared-Key [BAN89a]

*Message 2*  $S \rightarrow A : \{n_A, A \leftrightarrow^{k_{AB}} B, \text{fresh}(k_{AB}), \{A \leftrightarrow^{k_{AB}} B\}k_{BS}\}k_{AS}$  from  $S$

*Message 3*  $A \rightarrow B : \{A \leftrightarrow^{k_{AB}} B\}k_{BS}$  from  $S$

*Message 4*  $B \rightarrow A : \{n_B, A \leftrightarrow^{k_{AB}} B\}k_{AB}$  from  $B$

*Message 5*  $A \rightarrow B : \{n_B, A \leftrightarrow^{k_{AB}} B\}k_{AB}$  from  $A$

## NSKK Idealization

- First message is omitted
  - Plaintext is omitted
- It is assumed that principals recognize their own messages. Thus, with a shared key, if a recipient can decrypt a message, she can tell who it is from. As this is often implicitly clear, the *from* field is often omitted.
- What is inside the encrypted messages is also altered. Specifically, the key  $k_{AB}$  is replaced by assertions about it.
- Also in the last message  $n_B - 1$  is changed to just  $n_B$ .

## NSSK Initial State Assumptions

- P1.  $A$  believes  $A \leftrightarrow^{k_{AS}} S$**
- P2.  $B$  believes  $B \leftrightarrow^{k_{BS}} S$**
- P3.  $A$  believes  $S$  controls  $A \leftrightarrow^k B$**
- P4.  $B$  believes  $S$  controls  $A \leftrightarrow^k B$**
- P5.  $A$  believes  $S$  controls  $\text{fresh}(A \leftrightarrow^k B)$**
- P6.  $A$  believes  $\text{fresh}(n_A)$**
- P7.  $B$  believes  $\text{fresh}(n_B)$**

## NSSK Initial State Assumptions

- P1, P2 are beliefs in quality of long-term keys
  - S has similar beliefs but are not relevant
- P3, P4, P5 are jurisdiction beliefs
- P6, P7 are beliefs in freshness of each principal's nonces

## NSSK Annotated Protocol

**P8.** *A received  $\{n_A, A \leftrightarrow^{k_{AB}} B, \text{fresh}(k_{AB}), \{A \leftrightarrow^{k_{AB}} B\}_{k_{BS}}\}_{k_{AS}}$  from S*

**P9.** *B received  $\{A \leftrightarrow^{k_{AB}} B\}_{k_{BS}}$  from S*

**P10.** *A received  $\{n_B, A \leftrightarrow^{k_{AB}} B\}_{k_{AB}}$  from B*

**P11.** *B received  $\{nB, A \leftrightarrow^{k_{AB}} B\}_{k_{AB}}$  from A*

Basically read directly from idealized protocol

## NSSK Derivations

1. *A believes S said* ( $n_A, A \leftrightarrow^{k_{AB}} B, \text{fresh}(A \leftrightarrow^{k_{AB}} B), \{A \leftrightarrow^{k_{AB}} B\}_{k_{BS}}$ )  
By Message Meaning using P1, P8.
2. *A believes fresh* ( $n_A, A \leftrightarrow^{k_{AB}} B, \text{fresh}(A \leftrightarrow^{k_{AB}} B), \{A \leftrightarrow^{k_{AB}} B\}_{k_{BS}}$ )  
By Freshness Concatenation using 1, P6.
3. *A believes S believes* ( $n_A, A \leftrightarrow^{k_{AB}} B, \text{fresh}(A \leftrightarrow^{k_{AB}} B), \{A \leftrightarrow^{k_{AB}} B\}_{k_{BS}}$ )  
By Nonce Verification using 2, 1.
4. *A believes S believes* ( $A \leftrightarrow^{k_{AB}} B$ )  
By Belief Concatenation using 3.
5. *A believes S believes* ( $\text{fresh}(A \leftrightarrow^{k_{AB}} B)$ )  
By Belief Concatenation using 3.

## NSSK Derivations

6. *A believes* ( $A \leftrightarrow^{k_{AB}} B$ )  
By Jurisdiction using 4, P3.
7. *A believes fresh* ( $A \leftrightarrow^{k_{AB}} B$ )  
By Jurisdiction using 4, P5.

We have derived Alice's belief in the goodness and in the freshness of  $k_{AB}$ .  
How about Bob?

## NSSK Derivations

### 8. *B believes S said* ( $A \leftrightarrow^{k_{AB}} B$ )

By Message Meaning using P2, P9.

This gives us Bob's belief in the goodness of  $k_{AB}$ . Unlike Alice, Bob has sent no nonce at this point in the protocol. To get Bob's belief in freshness we need the following assumption.

### P12. *B believes fresh*( $A \leftrightarrow^{k_{AB}} B$ ) [Dubious]

This is different than P6, P7 which were based on nonces that the believing principal generates. Here Bob believes that a random value generated by someone else is fresh.

## NSSK Derivations

### 9. *B believes S believes* $A \leftrightarrow^{k_{AB}} B$

By Nonce Verification using P12, 8.

### 10. *B believes* $A \leftrightarrow^{k_{AB}} B$

By Jurisdiction using P4, 9.

## NSSK Derivations

**11.  $A$  believes  $B$  said  $(n_B, A \leftrightarrow^{kAB} B)$**

By Message Meaning using 6, P10.

**12.  $A$  believes fresh  $(n_B, A \leftrightarrow^{kAB} B)$**

By Freshness Conjunction using 7.

**13.  $A$  believes  $B$  believes  $(n_B, A \leftrightarrow^{kAB} B)$**

By Nonce Verification using 12, 11.

**14.  $A$  believes  $B$  believes  $A \leftrightarrow^{kAB} B$**

By Belief Conjunction using 13.

## NSSK Derivations

***Similarly we can get  $A$  believes  $B$  believes  $A \leftrightarrow^{kAB} B$***

By Belief Conjunction using 13.

See page 73, need clarification about use of nB

## NSSK: Denning-Sacco Attack [DS81]

Message 3  $E_A \rightarrow B : \{k_{AB}, A\}_{k_{BS}}$

Message 4  $B \rightarrow E_A : \{n_B\}_{k_{AB}}$

Message 5  $E_A \rightarrow B : \{n_B - 1\}_{k_{AB}}$

$E_A$  is the attacker masquerading as  $A$  using an old compromised session key  $k_{AB}$  within the lifetime of the long-term key  $k_{BS}$

The attack is not directly uncovered by BAN but BAN analysis shows the desired beliefs of  $B$  cannot be derived without the dubious assumption P12  $B$  believes  $\text{fresh}(A \leftrightarrow^{k_{AB}} B)$  that underlies this attack.

## The Nessett Protocol [Nes90]

Message 1  $A \rightarrow B : \{n_A, k_{AB}\}_{k_A^{-1}}$

Message 2  $B \rightarrow A : \{n_B\}_{k_{AB}}$

An obviously insecure protocol, yet proved "secure" using BAN

## The Nessett Protocol [Nes90]

### Idealized Nessett Protocol

Message 1  $A \rightarrow B : \{n_A, A \leftrightarrow^{k_{AB}} B\}_{k_A^{-1}}$

Message 2  $B \rightarrow A : \{A \leftrightarrow^{k_{AB}} B\}_{k_{AB}}$

### Annotation Premises

P1.  $B$  received  $\{n_A, A \leftrightarrow^{k_{AB}} B\}_{k_A^{-1}}$

P2.  $A$  received  $\{A \leftrightarrow^{k_{AB}} B\}_{k_{AB}}$

## The Nessett Protocol [Nes90]

### Initial State Assumptions

P3.  $B$  believes  $PK(k_A, A)$

P4.  $A$  believes  $A \leftrightarrow^{k_{AB}} B$

P5.  $A$  believes  $fresh(A \leftrightarrow^{k_{AB}} B)$

P6.  $B$  believes  $fresh(n_A)$

P7.  $B$  believes  $A$  controls  $(A \leftrightarrow^{k_{AB}} B)$

Note P6 whereby  $n_a$  is more naturally thought of as a timestamp rather than a nonce



## Nessett Protocol Derivations for Bob

1. *B believes A said* ( $n_A, A \leftrightarrow^{k_{AB}} B$ )  
By Message Meaning using P3, P1.
2. *B believes fresh*( $n_A, A \leftrightarrow^{k_{AB}} B$ )  
By Freshness Concatenation using P6.
3. *B believes A believes* ( $n_A, A \leftrightarrow^{k_{AB}} B$ )  
By Nonce Verification using 2, 1.
4. *B believes A believes A*  $\leftrightarrow^{k_{AB}} B$   
By Belief Concatenation using 3.
5. *B believes A*  $\leftrightarrow^{k_{AB}} B$   
By Jurisdiction using P7, 4.

## Nessett Protocol Derivations for Alice

6. *A believes B said*  $A \leftrightarrow^{k_{AB}} B$   
By Message Meaning using P4, P2.
7. *A believes B believes A*  $\leftrightarrow^{k_{AB}} B$   
By Nonce Verification using P5, 6.

These are Alice's second order beliefs in the goodness of  $k_{AB}$ .  
(Her first order belief was assumed.)

## The Nessel Protocol

- Nessel traces the source of the “flaw” to the scope of BAN. It addresses who gets and acknowledges a key (authentication), but it does not address who should not get a key (confidentiality).
- Burrows et al. respond to Nessel in [BAN90b] by noting that their paper explicitly limits discussion to authentication of honest principals. They explicitly do not attempt to detect unauthorized release of secrets.
- Alice’s action is inconsistent with meaning of  $A \text{ believes } A \leftrightarrow^{k_{AB}} B$ . What is needed is a way to reflect this mathematically. Suppose we could derive  $A \text{ believes } C \text{ has } k_{AB}$  (for arbitrary  $C$ ). Increasing expressiveness would let us formally demonstrate this.

## Beyond BAN

- GNY90
- AT91
- vO93
- And others
- SvO94, SvO96 unifies these