

# Logic of Authentication

## 1. BAN Logic

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## BAN Logic

- BAN is a logic of belief.
- In an analysis, the protocol is first idealized into messages containing assertions, then assumptions are stated, and finally conclusions are inferred based on the assertions in the idealized messages and those assumptions.

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## Source

These lectures are primarily based on:

- Paul Syverson and Iliano Cervesato, *The Logic of Authentication Protocols*, in R. Focardi, R. Gorrieri (Eds.): *Foundations of Security Analysis and Design, Lecture Notes in Computer Science, LNCS 2171*, Springer-Verlag 2001.

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## The language of BAN

- In all of these expressions,  $X$  is either a message or a formula.
- As we will see, every formula can be a message, but not every message is a formula.

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## Protocol 1 (Needham-Schroeder Shared-Key) [NS78]

*Message 1*  $A \rightarrow S : A, B, n_A$

*Message 2*  $S \rightarrow A : \{n_A, B, k_{AB}, \{k_{AB}, A\}_{k_{BS}}\}_{k_{AS}}$

*Message 3*  $A \rightarrow B : \{k_{AB}, A\}_{k_{BS}}$

*Message 4*  $B \rightarrow A : \{n_B\}_{k_{AB}}$

*Message 5*  $A \rightarrow B : \{n_B - 1\}_{k_{AB}}$

Nonces are random unpredictable values generated by a principal and included in messages so that she can tell any messages later received and containing her nonce must have been produced after she generated and sent the nonce.

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## The language of BAN

- $P$  believes  $X$  :
- $P$  received  $X$  : message;  
this may require decryption.
- $P$  said  $X$  :
- $P$  controls  $X$  :
- $\text{fresh}(X)$  : (Read 'X is fresh'.)  
 $X$  has not been sent in any message prior to current protocol run

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## The language of BAN

- $P \leftrightarrow^k Q$  : (Read ' $k$  is a good key for  $P$  and  $Q$ '.)  
 $k$  will never be discovered by any principal but  $P$ ,  $Q$ , or a principal trusted by  $P$  or  $Q$ . (The last case is necessary, since the server often sees, indeed generates,  $k$ .)
- $PK(P, k)$  : (Read ' $k$  is a public key of  $P$ '.)  
 The secret key,  $k^{-1}$ , corresponding to  $k$  will never be discovered by any principal but  $P$  or a principal trusted by  $P$ .
- $\{X\}k$  : Short for " $\{X\}k$  from  $P$ " (Read ' $X$  encrypted with  $k$  (from  $P$ ).')  
 This is the notation for encryption. Principals can recognize their own messages. Encrypted messages are uniquely readable and verifiable as such by holders of the right keys.

## BAN Rules: Nonce Verification

$$\frac{P \text{ believes } \text{fresh}(X) \quad P \text{ believes } Q \text{ said } X}{P \text{ believes } Q \text{ believes } X}$$

This rule allows promotion from the past to the present (something said some time in the past to a present belief).  
 In order to be applied,  $X$  should not contain any encrypted text.

## BAN Rules: Message Meaning

$$\frac{P \text{ believes } P \leftrightarrow^k Q \quad P \text{ received } \{X\}k}{P \text{ believes } Q \text{ said } X}$$

"If  $P$  receives  $X$  encrypted with  $k$  and if  $P$  believes  $k$  is a good key for talking with  $Q$ , then  $P$  believes  $Q$  once said  $X$ ."

In applying symmetric keys, there is no explicit distinction between signing and encryption.

## BAN Rules: Jurisdiction

$$\frac{P \text{ believes } Q \text{ controls } X \quad P \text{ believes } Q \text{ believes } X}{P \text{ believes } X}$$

The jurisdiction rule allows inferences that a principal believes a key is good, even though it is a random string that he has never seen before.

## BAN Rules: Message Meaning

$$\frac{P \text{ believes } PK(Q, k) \quad P \text{ received } \{X\}k^{-1}}{P \text{ believes } Q \text{ said } X}$$

There is no explicit distinction between signing and encryption. Both are represented by  $\{X\}k$  or  $\{X\}k^{-1}$ . The distinction is implicit in the notation for the key used:  $k$  or  $k^{-1}$ .

## BAN Rules: Belief Concatenation

$$\frac{P \text{ believes } X \quad P \text{ believes } Y}{P \text{ believes } (X, Y)}$$

The obvious rules apply to beliefs concerning concatenations of messages/conjunctions of formulae.  
 Concatenations of messages and conjunctions of formulae are both represented as  $(X, Y)$  in the above rules.

## BAN Rules: Belief Conjunction

$$\frac{P \text{ believes } Q \text{ believes } (X, Y)}{P \text{ believes } Q \text{ believes } X} \quad \frac{P \text{ believes } Q \text{ said } (X, Y)}{P \text{ believes } Q \text{ said } X}$$

We do not list all of the rules; we give only a representative sampling.

## BAN Protocol Analysis

1. Idealize the protocol.
2. Write assumptions about the initial state.
3. Annotate the protocol: For each message transmission  $P \rightarrow Q : M$  in the protocol, assert  $Q \text{ received } M$ .
4. Use the logic to derive the beliefs held by protocol principals.

## BAN Rules: Freshness Conjunction

$$\frac{P \text{ believes fresh}(X)}{P \text{ believes fresh}(X, Y)}$$

For some inexplicable reason, this is a commonly misunderstood BAN rule. Some try to deny it; others try to assert the converse rule. Be wary of these mistakes.

## Protocol 1 (Needham-Schroeder Shared-Key) [NS78]

Message 1  $A \rightarrow S : A, B, n_A$

Message 2  $S \rightarrow A : \{n_A, B, k_{AB}, \{k_{AB}, A\}_{k_{BS}}\}_{k_{AS}}$

Message 3  $A \rightarrow B : \{k_{AB}, A\}_{k_{BS}}$

Message 4  $B \rightarrow A : \{n_B\}_{k_{AB}}$

Message 5  $A \rightarrow B : \{n_B - 1\}_{k_{AB}}$

## BAN Rules: Receiving Rules: Seeing is Receiving

$$\frac{P \text{ believes } P \leftrightarrow^* Q}{P \text{ received } \{X\}_k} \quad \frac{P \text{ received } (X, Y)}{P \text{ received } X}$$

A principal receiving a message also receives submessages he can uncover.

## Idealized Needham-Schroeder Shared-Key [BAN89a]

Message 2  $S \rightarrow A : \{n_A, A \leftrightarrow^{k_{AB}} B, \text{fresh}(k_{AB}), \{A \leftrightarrow^{k_{AB}} B\}_{k_{BS}}\}_{k_{AS}} \text{ from } S$

Message 3  $A \rightarrow B : \{A \leftrightarrow^{k_{AB}} B\}_{k_{BS}} \text{ from } S$

Message 4  $B \rightarrow A : \{n_B, A \leftrightarrow^{k_{AB}} B\}_{k_{AB}} \text{ from } B$

Message 5  $A \rightarrow B : \{n_B, A \leftrightarrow^{k_{AB}} B\}_{k_{AB}} \text{ from } A$

## NSSK Idealization

- First message is omitted
  - Plaintext is omitted
- It is assumed that principals recognize their own messages. Thus, with a shared key, if a recipient can decrypt a message, she can tell who it is from. As this is often implicitly clear, the *from* field is often omitted.
- What is inside the encrypted messages is also altered. Specifically, the key  $k_{AB}$  is replaced by assertions about it.
- Also in the last message  $n_B - 1$  is changed to just  $n_B$ .

## NSSK Annotated Protocol

**P8.** *A received*  $\{n_A, A \leftrightarrow^{k_{AB}} B, \text{fresh}(k_{AB}), \{A \leftrightarrow^{k_{AB}} B\}k_{BS}\}k_{AS}$  from *S*

**P9.** *B received*  $\{A \leftrightarrow^{k_{AB}} B\}k_{BS}$  from *S*

**P10.** *A received*  $\{n_B, A \leftrightarrow^{k_{AB}} B\}k_{AB}$  from *B*

**P11.** *B received*  $\{n_B, A \leftrightarrow^{k_{AB}} B\}k_{AB}$  from *A*

Basically read directly from idealized protocol

## NSSK Initial State Assumptions

- P1.** *A believes*  $A \leftrightarrow^{k_{AS}} S$
- P2.** *B believes*  $B \leftrightarrow^{k_{BS}} S$
- P3.** *A believes* *S controls*  $A \leftrightarrow^k B$
- P4.** *B believes* *S controls*  $A \leftrightarrow^k B$
- P5.** *A believes* *S controls*  $\text{fresh}(A \leftrightarrow^k B)$
- P6.** *A believes*  $\text{fresh}(n_A)$
- P7.** *B believes*  $\text{fresh}(n_B)$

## NSSK Derivations

- A believes* *S said*  $(n_A, A \leftrightarrow^{k_{AB}} B, \text{fresh}(A \leftrightarrow^{k_{AB}} B), \{A \leftrightarrow^{k_{AB}} B\}k_{BS})$   
By Message Meaning using P1, P8.
- A believes*  $\text{fresh}(n_A, A \leftrightarrow^{k_{AB}} B, \text{fresh}(A \leftrightarrow^{k_{AB}} B), \{A \leftrightarrow^{k_{AB}} B\}k_{BS})$   
By Freshness Concatenation using 1, P6.
- A believes* *S believes*  $(n_A, A \leftrightarrow^{k_{AB}} B, \text{fresh}(A \leftrightarrow^{k_{AB}} B), \{A \leftrightarrow^{k_{AB}} B\}k_{BS})$   
By Nonce Verification using 2, 1.
- A believes* *S believes*  $(A \leftrightarrow^{k_{AB}} B)$   
By Belief Concatenation using 3.
- A believes* *S believes*  $(\text{fresh}(A \leftrightarrow^{k_{AB}} B))$   
By Belief Concatenation using 3.

## NSSK Initial State Assumptions

- P1, P2 are beliefs in quality of long-term keys
  - S has similar beliefs but are not relevant
- P3, P4, P5 are jurisdiction beliefs
- P6, P7 are beliefs in freshness of each principal's nonces

## NSSK Derivations

- A believes*  $(A \leftrightarrow^{k_{AB}} B)$   
By Jurisdiction using 4, P3.
- A believes*  $\text{fresh}(A \leftrightarrow^{k_{AB}} B)$   
By Jurisdiction using 4, P5.

We have derived Alice's belief in the goodness and in the freshness of  $k_{AB}$ .  
How about Bob?

## NSSK Derivations

### 8. $B$ believes $S$ said $(A \leftrightarrow^{k_{AB}} B)$

By Message Meaning using P2, P9.

This gives us Bob's belief in the goodness of  $k_{AB}$ . Unlike Alice, Bob has sent no nonce at this point in the protocol. To get Bob's belief in freshness we need the following assumption.

### P12. $B$ believes fresh $(A \leftrightarrow^{k_{AB}} B)$ [Dubious]

This is different than P6, P7 which were based on nonces that the believing principal generates. Here Bob believes that a random value generated by someone else is fresh.

## NSSK Derivations

Similarly we can get  $A$  believes  $B$  believes  $A \leftrightarrow^{k_{AB}} B$

By Belief Concatenation using 13.

See page 73, need clarification about use of nB

## NSSK Derivations

### 9. $B$ believes $S$ believes $A \leftrightarrow^{k_{AB}} B$

By Nonce Verification using P12, 8.

### 10. $B$ believes $A \leftrightarrow^{k_{AB}} B$

By Jurisdiction using P4, 9.

## NSSK: Denning-Sacco Attack [DS81]

Message 3  $E_A \rightarrow B : \{k_{AB}, A\}_{k_{BS}}$

Message 4  $B \rightarrow E_A : \{n_B\}_{k_{AB}}$

Message 5  $E_A \rightarrow B : \{n_B - 1\}_{k_{AB}}$

$E_A$  is the attacker masquerading as  $A$  using an old compromised session key  $k_{AB}$  within the lifetime of the long-term key  $k_{BS}$

The attack is not directly uncovered by BAN but BAN analysis shows the desired beliefs of  $B$  cannot be derived without the dubious assumption P12  $B$  believes fresh  $(A \leftrightarrow^{k_{AB}} B)$  that underlies this attack.

## NSSK Derivations

### 11. $A$ believes $B$ said $(n_B, A \leftrightarrow^{k_{AB}} B)$

By Message Meaning using 6, P10.

### 12. $A$ believes fresh $(n_B, A \leftrightarrow^{k_{AB}} B)$

By Freshness Concatenation using 7.

### 13. $A$ believes $B$ believes $(n_B, A \leftrightarrow^{k_{AB}} B)$

By Nonce Verification using 12, 11.

### 14. $A$ believes $B$ believes $A \leftrightarrow^{k_{AB}} B$

By Belief Concatenation using 13.

## The Nessett Protocol [Nes90]

Message 1  $A \rightarrow B : \{n_A, k_{AB}\}_{k_A^{-1}}$

Message 2  $B \rightarrow A : \{n_B\}_{k_{AB}}$

An obviously insecure protocol, yet proved "secure" using BAN

## The Nessett Protocol [Nes90]

### Idealized Nessett Protocol

Message 1  $A \rightarrow B : \{n_A, A \leftrightarrow^{k_{AB}} B\}_{k_A^{-1}}$

Message 2  $B \rightarrow A : \{A \leftrightarrow^{k_{AB}} B\}_{k_{AB}}$

### Annotation Premises

P1.  $B$  received  $\{n_A, A \leftrightarrow^{k_{AB}} B\}_{k_A^{-1}}$

P2.  $A$  received  $\{A \leftrightarrow^{k_{AB}} B\}_{k_{AB}}$

## Nessett Protocol Derivations for Alice

6.  $A$  believes  $B$  said  $A \leftrightarrow^{k_{AB}} B$

By Message Meaning using P4, P2.

7.  $A$  believes  $B$  believes  $A \leftrightarrow^{k_{AB}} B$

By Nonce Verification using P5, 6.

These are Alice's second order beliefs in the goodness of  $k_{AB}$ .  
(Her first order belief was assumed.)

## The Nessett Protocol [Nes90]

### Initial State Assumptions

P3.  $B$  believes  $PK(k_A, A)$

P4.  $A$  believes  $A \leftrightarrow^{k_{AB}} B$

P5.  $A$  believes  $fresh(A \leftrightarrow^{k_{AB}} B)$

P6.  $B$  believes  $fresh(n_A)$

P7.  $B$  believes  $A$  controls  $(A \leftrightarrow^{k_{AB}} B)$

Note P6 whereby  $n_A$  is more naturally thought of as a timestamp rather than a nonce

## The Nessett Protocol

- Nessett traces the source of the "flaw" to the scope of BAN. It addresses who gets and acknowledges a key (authentication), but it does not address who should not get a key (confidentiality).
- Burrows et al. respond to Nessett in [BAN90b] by noting that their paper explicitly limits discussion to authentication of honest principals. They explicitly do not attempt to detect unauthorized release of secrets.
- Alice's action is inconsistent with meaning of  $A$  believes  $A \leftrightarrow^{k_{AB}} B$ . What is needed is a way to reflect this mathematically. Suppose we could derive  $A$  believes  $C$  has  $k_{AB}$  (for arbitrary  $C$ ). Increasing expressiveness would let us formally demonstrate this.

## Nessett Protocol Derivations for Bob

1.  $B$  believes  $A$  said  $(n_A, A \leftrightarrow^{k_{AB}} B)$

By Message Meaning using P3, P1.

2.  $B$  believes  $fresh(n_A, A \leftrightarrow^{k_{AB}} B)$

By Freshness Conjunction using P6.

3.  $B$  believes  $A$  believes  $(n_A, A \leftrightarrow^{k_{AB}} B)$

By Nonce Verification using 2, 1.

4.  $B$  believes  $A$  believes  $A \leftrightarrow^{k_{AB}} B$

By Belief Conjunction using 3.

5.  $B$  believes  $A \leftrightarrow^{k_{AB}} B$

By Jurisdiction using P7, 4.

## Beyond BAN

- GNY90
- AT91
- vO93
- And others
- SvO94, SvO96 unifies these